Spotlight 3.2

Choosing an inequality index

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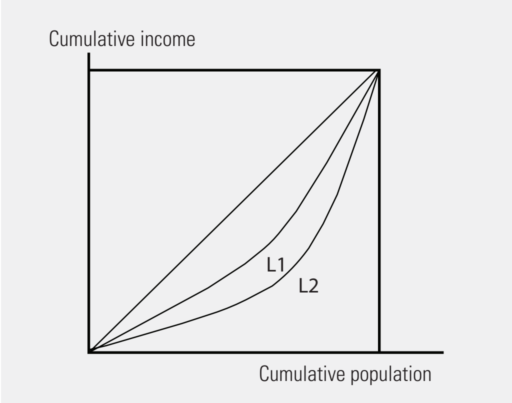
A useful way to describe the distribution of income is the Lorenz curve, constructed as follows.[1] First, the population is ranked according to income (or consumption, wealth or another measure of resources) from the lowest to the highest. Then the cumulative shares of individuals in the population are plotted against their respective cumulative share in total income. The curve drawn is called the Lorenz curve. The horizontal axis of the Lorenz curve shows the cumulative percentages of the population arranged in increasing order of income. The vertical axis shows the percentage of total income received by a fraction of the population. For example, the (80 percent, 60 percent) point on the Lorenz curve means that the poorest 80 percent of the population receives 60 percent of total income while the richest 20 percent receives 40 percent of total income.[2]

Figure S3.2.1 shows two Lorenz curves: L_1 and L_2. If everybody has the same income, the Lorenz curve will coincide with the 45-degree line. The greater the level of inequality, the farther the Lorenz curve will be from the 45-degree line. In the figure, L_2 lies below and to the right of L_1, so an inequality index would be expected to indicate greater inequality in the L_2 case. Another way to see this is that the poorest *x* percent of the population will always have an equal or greater share of income under L_1 than under L_2, regardless of what *x* is. This is called the Lorenz dominance criterion or Lorenz criterion for short.

FIGURE S3.1.1

Lorenz curve



Source: Authors' creation.

What constitutes a "good" inequality index? One approach is to require the measure to be consistent with the Lorenz criterion: that is, to be Lorenz consistent. For a measure to be Lorenz consistent the following two conditions must hold: First, inequality rises (declines) when the Lorenz curve lies everywhere below (above) the original Lorenz curve as with L_2 compared with L_1 (L_1 compared to L_2) in the figure. Second, inequality is the same when Lorenz curves are identical. For a measure to be Weakly Lorenz Consistent, condition 1 becomes the following: 1'. inequality rises (declines) or stays the same when the Lorenz curve lies everywhere below (above) the original Lorenz curve.

A second approach is to require the inequality index to fulfil the following four principles:

1 Symmetry (or anonymity). If two people switch incomes, the index level should not change.
2 Population invariance (or replication invariance). If the population is replicated or "cloned" one or more times, the index level should not change.
3 Scale invariance (or mean independence). If all incomes are scaled up or down by a common factor (for example, doubled), the index level should not change.
4 Transfer (or the Pigou-Dalton Transfer Principle). If income is transferred from one person to another who is richer, the index level should increase. In other words, in the face of a regressive transfer, the index level must rise.

It can be shown that indices satisfying these four principles are Lorenz Consistent and vice versa.

These indices include:

- **Summary indices** based on relatively complex formulas designed to capture inequality along the entire distribution. The most commonly used are (in alphabetical order): the Atkinson, Gini and Theil measures (and the generalized entropy measures, more generally).

While inequality measures that satisfy the transfer principle are in common use, there are also simpler indices that do not satisfy 1–4 but are popular. These include:

- **Partial indices** based on simple formulas that focus on inequality across certain parts of the distribution. These include the Kuznets ratios expressed as the income share of top *x* percent over the income share of bottom *y* percent. There are, of course, many possible Kuznets ratios. The one proposed by the Nobel Laureate Simon Kuznets was 20/40.[3] Partial indices also include the top income shares, expressed as the income share of the top *x* percent. Common examples include the income share of the top 1 percent or of the top 10 percent.[4] The top income shares are, in fact, limiting cases of Kuznets ratios obtained by setting the "bottom" income share to cover the entire population: that is, by setting *y* percent = 100 percent.[5]

Such partial Indices satisfy the following principle:

4' Weak transfer principle: If income is transferred from one person to another who is richer (or equally rich), the index level should increase or remain unchanged.

In other words, in the face of a regressive transfer, the inequality index can never decline, but it may remain unchanged. It can be shown that indices satisfying 1–3 and 4' principles are weakly Lorenz consistent and vice versa.

In sum, the summary indices of Atkinson, Gini and Theil (and the whole family of Generalized Entropy Indices) satisfy principles 1–3 and 4 and thus are Lorenz consistent (and vice versa). This guarantees that in the face of a regressive (progressive) transfer anywhere along the distribution, inequality measured by any of these indices will rise (decline). In contrast, the Kuznets ratios and top income shares focus on limited ranges of incomes and thus violate the transfer principle (and thus violate Lorenz consistency). The latter means that transfers entirely within or entirely outside the relevant ranges have no effect on measured inequality. For example, the 10/40 ratio is insensitive to regressive transfers that stay within the poorest 40 percent, within the richest 10 percent or within the remaining 50 percent in the middle, while the income share of the top 1 percent is insensitive to transfers within the top 1 percent and within the bottom 99 percent. Despite disagreeing with the transfer principle, and thus the Lorenz criterion, these partial indices are useful for conveying easily understood information about the extent of inequality. Importantly, they satisfy the weak transfer principle and thus guarantee that in the face of a regressive transfer anywhere along the distribution, inequality measured by any of these indices will never decline but, notably, it can stay the same.

In contrast, other common inequality indices do not even fulfil the weak transfer principle (transfer principle 4'). Examples include the quantile ratios (such as the income of percentile 90 to the income of the 10th percentile also known as the p90/p10 ratio) and the variance of logarithms. For example, a transfer from the 5th percentile to the 10th would reduce the p90/p10 ratio despite the fact that the transfer is clearly regressive because it redistributes income from the very poor to the less poor. Regressive transfers at the upper end of the distribution can lower the variance of logarithms and lead to extreme conflicts with the Lorenz criterion.[6]

Finally, the mean to median ratio (mean divided by the median) is a measure of skewness that can also be interpreted as a partial index of inequality. Virtually every inequality measure is a ratio of two "income standards" that summarize the size of the income distributions from two perspectives: one that emphasizes higher incomes and a second that emphasizes lower incomes.[7] So long as only distributions that are skewed to the right are considered, the mean exceeds the median, and the mean to median ratio takes on this form. This index satisfies the first three principles but can violate the weak transfer principle when the regressive transfer raises the median income. Like the other partial indices, it is weaker in terms of the properties it satisfies but has the advantage of simplicity and is often used in political economy.[8]

How to apply the above in practice? When making pairwise comparisons, first graph the Lorenz curves. If the Lorenz curves do not cross, an unambiguous Lorenz comparison can be made. One can conclude from this that any reasonable (that is, Lorenz consistent) measure would agree that inequality has unambiguously increased or declined, according to what the Lorenz curves indicates. However, it is also possible that the Lorenz curves cross, in which case reasonable inequality measures can disagree. What can be done when Lorenz curves cross? One approach is to narrow the set of reasonable inequality measures using an additional criterion. For instance, transfer-sensitive measures are Lorenz consistent measures that emphasize distributional changes at the lower end over those at the upper end. The Atkinson class and the two Theil measures (including the mean log deviation) are transfer-sensitive measures. By contrast, the coefficient of variation (standard deviation divided by the mean) is neutral with respect to where transfers occur, while many other generalized entropy measures emphasize distributional changes at the upper end and thus are not in the set of transfer-sensitive measures.

When do all transfer-sensitive measures agree? As a subset of Lorenz-consistent measures, they agree when Lorenz curves do not cross as well as in many cases when they do cross. For example, suppose that Lorenz curves cross once and that the first Lorenz curve is higher at lower incomes than the second. There is a simple test: The first has less inequality than the second, according to all transfer-sensitive measures exactly when the coefficient of variation for the first is no higher than that for the second.[9] An even simpler approach is to select a (finite) set of particularly relevant inequality measures for making inequality comparisons. If all agree on a given comparison, the result is robust. If not, the conclusion is ambiguous for that set of measures, with inequality ranked one way for some measures and reversed for others.

Table S3.2.1 shows the statistics most frequently published in commonly used international databases.[9]

TABLE S3.2.1

Statistics most frequently published in 10 commonly used international databases

Statistic	Frequency
Gini	9
Quantile ratio 90/10	4
Theil	3
Top 10 percent	3

Source: Authors' creation.

Thus, the most frequently reported inequality measures include two that are Lorenz consistent (the Gini and Theil measures), one that is weakly Lorenz consistent (the top 10 percent) and one that is neither (the 90/10 quantile ratio). In addition to inequality measures, international datasets report other statistics. Among those, the most frequent is the distribution of income by decile.[10]

Notes

1 Named after Max Otto Lorenz, a US economist who developed the idea of the Lorenz curve in 1905.
2 Often, especially with historical data, we only have grouped-data or information on equal-sized population groups such as quintiles or deciles (5 or 10 groups, respectively). The resulting Lorenz curve is an approximation of the actual Lorenz curve where inequality within each group has been suppressed.
3 Some international databases report the 20/20 (sometimes called S80/S20) and 10/40 ratios.
4 The top 1 percent has been the focus of the recent literature on top incomes. See, for example, Atkinson, Piketty and Saez (2011).
5 By definition, 100 percent of the population receives 100 percent of the income so the denominator of the Kuznets ratio becomes 100/100 = 1, and thus the 1/100 Kuznets ratio equals 1 percent.
6 Foster and Ok 1999.
7 Foster and others (2013, p. 15). For example, one Atkinson measure compares the higher arithmetic mean to the lower geometric means; the 1 percent income share effectively compares the higher 1 percent mean to the lower arithmetic mean.
8 The mean to median ratio is the inequality measure used by Meltzer and Richards (1981) in their model to predict the size of government. The greater the ratio, the higher the taxes and redistribution.
9 For details, see Shorrocks and Foster (1987). See also Zheng (2018), who presents additional criteria for making comparisons when Lorenz curves cross.
10 The complete set of measures reported in international databases and their properties can be found in supplemental material for this spotlight available at http://hdr.undp.org/en/2019-report.